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Prediction of thermal conductivities of laminated composites using penny-shaped fillers †

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Abstract

A new model is proposed to predict the thermal conductivities of laminated composites, where the Eshelby method modified with Mori-Tanaka's mean field approach is employed to consider the interaction effect. Based on the equivalency of composites with penny-shaped fillers and composites with layers of components, each lamina is considered as a penny-shaped filler and its thermal conductivities are computed by modified Eshelby method. The laminated composites are then simulated as the matrix and penny-shaped fillers of different thermal conductivities. By comparing the results of the laminated composites predicted by the present model and conventional approach combined with the potential theory and electrical analogy, the applicability of the present model to predict the thermal conductivities of the laminated composites is validated.

Keywords: Eshelby model; Laminated composites; Mori-Tanaka's mean field approach; Penny-shaped filler; Thermal conductivity

1. Introduction

The thermal conductivity of composite materials is of importance in many applications with high in-use temperatures. A number of analytical models for predicting the thermal conductivity of the composite materials have been proposed [1-15], whose derivations are based on the physical structures and constituent thermal conductivities of the composite materials. Fillers in the composites are mainly divided into two groups such as continuous and discontinuous shapes, based on which the models for the prediction of the thermal conductivity are quite different.

The modified Eshelby method (MEM) [16, 17] considering the interactions between the fillers has the advantage of the applicability to predict the thermal conductivity of the composites with various shapes of discontinuous fillers, so it has been extensively used for short fiber or particulate composites [1, 2, 5, 7]. Hatta et al. [5] predicted the thermal conductivities of composite materials with various shapes of fillers such as a particle, short fiber, flake, and whisker using MEM and the predicted results were compared with the measured results. On the other hand, laminated composites are representative composites with the continuous fillers, whose thermal conductivities have not been predicted by MEM. The most conventional approach for predicting their thermal conductivity is as follows [6, 8, 15]. The thermal conductivities of a unidirectional lamina in the longitudinal (filler) and its perpendicular directions are first computed. The longitudinal thermal conductivity k_L is obtained from the rule of mixtures, while the transverse thermal conductivity k_T is derived from an effective-medium approach [18] analogous to the self-consistent method. The thermal conductivities of the laminated composites are then predicted by using an electrical analogy [6, 8, 10, 15].

Many researches have been made on penny-shaped filler problems using the Eshelby method since

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Eshelby derived the Eshelby tensor through imaginary cut and welding processes. Some of them are related to the derivation of the Eshelby tensor [19, 20], while others are focused on the prediction of elastic moduli [21-26] and fracture analysis [27-32] of materials with penny-shaped cracks by using Eshelby's equivalent inclusion method. Although a wide range of analytical studies have been done for composites with penny-shaped fillers, a thermal problem for these composites has not been examined by MEM [16, 17]. In the present study, the analytical model for predicting the thermal conductivities of the laminated composites is proposed using the penny-shaped fillers, where the Eshelby method [16] with Mori-Tanaka's mean field approach [17] is employed for considering the interaction effect between the fillers. The closed-form solutions of the thermal conductivities by the present model are compared with the results for the composites with layers of the components by the series and parallel models, through which their results are proved to be the same for a special case of the penny-shaped filler. Based on this structural equivalency, the laminated composites can be simulated as a matrix involving the penny-shaped fillers of different thermal conductivities, which are then converted into the multi-phase composites. By computing the thermal conductivities of unidirectional laminae with misoriented fillers with MEM and then embedding them into the matrix, the thermal conductivities of the laminated composites are predicted to be compared and discussed with the results computed by the conventional approach [6, 8, 15].

2. Analytical models

2.1 Modified Eshelby model for composites with misaligned fillers

Let's consider the infinite domain of a composite (D) in which fillers (Ω) of any shape are randomly distributed in the $x_1 - x_3$ plane with an orientation angle θ relative to the x_3 axis of the global coordinate system, as shown in Fig. 1(a). The local coordinate system is set to the axes of the filler. Fig. 1(a) is converted into Fig. 1(b) by using Eshelby's equivalent inclusion method. The matrix and filler are assumed to be isotropic, whose thermal conductivities are denoted by k_m and k_f, respectively. Subscripts *m* and *f* represent the matrix and filler, respectively, and bold-faced letters denote a vector or matrix. The prime and the unprimed indicate the quantities referred to the local and the global coordinate system, respectively.

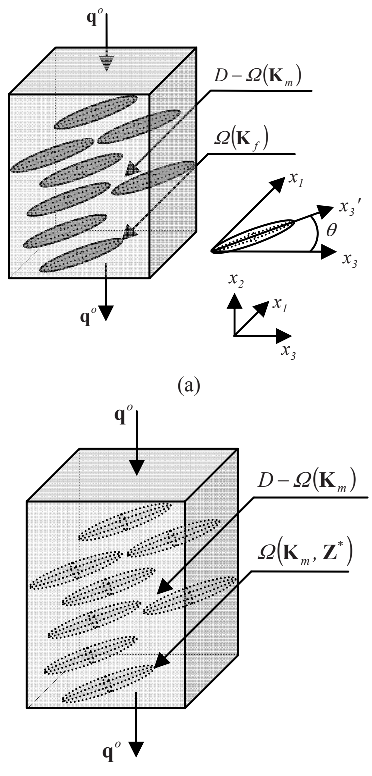

Fig. 1. Analytical model for computing heat fluxes and temperature gradient vector in both filler and matrix: (a) original problem, which is converted to (b) Eshelby's equivalent inclusion problem. Fillers have an orientation angle of θ .

The composite is subjected to a constant heat flux ($\mathbf{q}^o$) along the global coordinate system. By applying MEM [16, 17] to the composite, heat fluxes in the matrix and filler domain, $\mathbf{q}_m$ and $\mathbf{q}_f$, are expressed as

$$\mathbf{q}_f = -\mathbf{K}_f\left(\mathbf{Z}^o + \overline{\mathbf{Z}} + \mathbf{Z}\right) = -\mathbf{K}_m\left(\mathbf{Z}^o + \overline{\mathbf{Z}} + \mathbf{Z} - \mathbf{Z}^*\right), \quad (1)$$

$$\mathbf{q}_m = -\mathbf{K}_m\left(\mathbf{Z}^o + \overline{\mathbf{Z}}\right), \quad (2)$$

where $\mathbf{K}$ is the thermal conductivity matrix, $\overline{\mathbf{Z}}$ is the average of a disturbed temperature gradient vector in the matrix, $\mathbf{Z}$ is the disturbed temperature-gradient vector in the filler, $\mathbf{Z}^*$ is the eigentemperature-gradient vector of the equivalent inclusion problem. Since $\mathbf{Z}^o$ is the constant temperature gradient vector in the matrix without any filler generated by the heat flux $\mathbf{q}^o$, their relationship is given by

$$\mathbf{q}^o = -\mathbf{K}_m \mathbf{Z}^o . \quad (3)$$

Since the integration of the disturbed heat flux over the entire composite domain is reduced to zero, $\bar{\mathbf{Z}}$ is obtained from Eqs. (1)-3) as

$$\bar{\mathbf{Z}} + f\left(\mathbf{Z} - \mathbf{Z}^*\right) = 0 , \quad (4)$$

where f is the filler volume fraction. From Eqs. (1), (2), and (4), the eigentemperature-gradient vector $\mathbf{Z}^*$ is derived as

$$\left(\mathbf{K}_f - \mathbf{K}_m\right)\left[(1-f)\mathbf{Z} + f\mathbf{Z}^*\right] + \mathbf{K}_m\mathbf{Z}^* = \left(\mathbf{K}_m - \mathbf{K}_f\right)\mathbf{Z}^o \quad (5)$$

Since $\mathbf{Z}'$ is related to $\mathbf{Z}'^*$ through Eshelby's tensor $\mathbf{S}$ as

$$\mathbf{Z}' = \mathbf{S}\mathbf{Z}'^* . \quad (6)$$

From Eqs. (5) and (6) and the transformation matrix $\mathbf{X}$, $\mathbf{Z}'^*$ is represented as

$$\begin{aligned}&\left\{\left(\mathbf{K}_f - \mathbf{K}_m\right)\left[(1-f)\mathbf{X}\mathbf{S} + f\mathbf{X}\right] + \mathbf{K}_m\mathbf{X}\right\}\mathbf{Z}'^* \\ &= \left(\mathbf{K}_m - \mathbf{K}_f\right)\mathbf{Z}^o\end{aligned} , \quad (7)$$

and further simplified as

$$\mathbf{Z}'^* = \mathbf{T}\left(\mathbf{K}_m - \mathbf{K}_f\right)\mathbf{Z}^o , \quad (8)$$

where $\mathbf{T}$ is defined as

$$\mathbf{T} = \left\{\left(\mathbf{K}_f - \mathbf{K}_m\right)\left[(1-f)\mathbf{X}\mathbf{S} + f\,\mathbf{X}\right] + \mathbf{K}_m\mathbf{X}\right\}^{-1} . \quad (9)$$

The total temperature gradient vectors in the matrix and filler, $\mathbf{Z}_m$ and $\mathbf{Z}_f$, are defined as

$$\mathbf{Z}_m = \mathbf{Z}^o + \bar{\mathbf{Z}} , \quad (10)$$

$$\mathbf{Z}_f = \mathbf{Z}^o + \bar{\mathbf{Z}} + \mathbf{Z} . \quad (11)$$

Their volume average in the entire composite is equal to the temperature gradient vector in the composite $\mathbf{Z}_c$, which is given by

$$\mathbf{Z}_c = (1-f)\mathbf{Z}_m + f\,\mathbf{Z}_f = \mathbf{Z}^o + f\,\mathbf{Z}^* . \quad (12)$$

By replacing $\mathbf{Z}^*$ with $\mathbf{Z}'^*$ in Eq. (12), $\mathbf{Z}_c$ takes the form

$$\mathbf{Z}_c = \left[\mathbf{I} + f\,\mathbf{X}\mathbf{T}\left(\mathbf{K}_m - \mathbf{K}_f\right)\right]\mathbf{Z}^o , \quad (13)$$

where $\mathbf{I}$ is the 3×3 identity matrix. Since the composite is subject to the heat flux $\mathbf{q}^o$, the relationship between the heat flux, temperature gradient, and thermal conductivity of the composite is expressed as

$$\mathbf{q}_c = -\mathbf{K}_c\mathbf{Z}_c = \mathbf{q}^o . \quad (14)$$

Finally, the thermal conductivity of the composite $\mathbf{K}_c$ in the global coordinate system is determined by Eqs. (3), (13), and (14), which is reduced to

$$\mathbf{K}_c = \mathbf{K}_m\left[\mathbf{I} + f\mathbf{X}\mathbf{T}\left(\mathbf{K}_m - \mathbf{K}_f\right)\right]^{-1} . \quad (15)$$

2.2 Modified Eshelby model for thermal conductivities of laminated composites

A laminated composite is composed of a stack of N layers, and their thermal conductivities are different from each other and computed from Eq. (15) by considering the layer as a composite with misoriented continuous fillers. Each layer can be simulated as a penny-shaped filler, which will be explained later. To compute the thermal conductivities of the laminated composite, all layers are considered to be penny-shaped fillers and embedded into the matrix which is equivalent to the matrix material of each layer, as shown in Fig. 2. The problem is then converted into a multi-phase composite.

The procedures hereafter are the same as those mentioned in section 2.1. By using Eshelby's equivalent inclusion method [16] with Mori-Tanaka's mean field approach [17], the heat fluxes in the matrix and the k-th layer are expressed as

$$\begin{aligned}\mathbf{q}_f^k &= -\mathbf{K}_f^k\left(\mathbf{Z}^o + \bar{\mathbf{Z}} + \mathbf{Z}^k\right) \\ &= -\mathbf{K}_m\left(\mathbf{Z}^o + \bar{\mathbf{Z}} + \mathbf{Z}^k - \mathbf{Z}^{*k}\right)\end{aligned} , \quad (16)$$

where the superscript k means the k-th layer. By analogy with Eq. (4), $\bar{\mathbf{Z}}$ is given by

$$\bar{\mathbf{Z}} + \sum_{k=1}^{N} f_k\left(\mathbf{Z}^k - \mathbf{Z}^{*k}\right) = 0 , \quad (17)$$

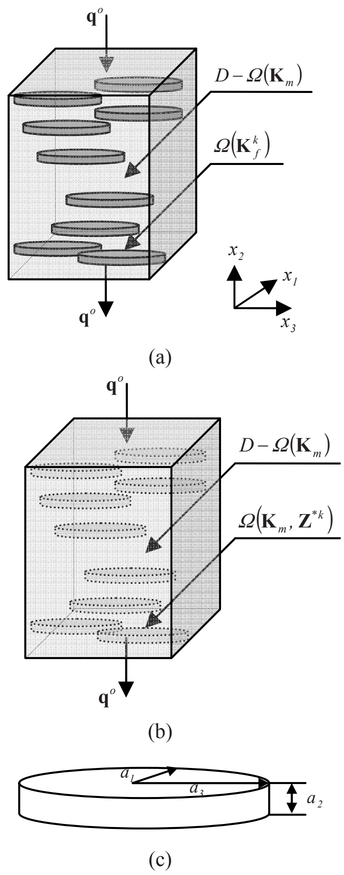

Fig. 2. An analytical model for composites with multi-phase fillers of penny shape: (a) original problem, which is converted to (b) Eshelby's equivalent inclusion problem, and (c) the shape of a penny.

where f_k represents the filler volume fraction of the k-th layer. f is the sum of f_k and equal to 1. By inserting Eq. (17) into Eq. (16), the relationship between eigen-temperature gradient vectors is derived as

$$\left\{\left(\mathbf{K}_f^k-\mathbf{K}_m\right)\left[\left(1-f_k\right)\mathbf{S}+f_k\mathbf{I}\right]+\mathbf{K}_m\right\}\mathbf{Z}^{*k} +\left(\mathbf{K}_m-\mathbf{K}_f^k\right)\sum_{\substack{i=1\\i\neq k}}^{N} f_i\left(\mathbf{S}-\mathbf{I}\right)\mathbf{Z}^{*i}=\left(\mathbf{K}_m-\mathbf{K}_f^k\right)\mathbf{Z}^o, \quad (18)$$

where $\mathbf{I}$ denotes the identity matrix and $\mathbf{S}$ is Eshelby tensor for the penny shape. The relation is simply restated as

$$\mathbf{A}_{k1}\mathbf{Z}^{*1}+\mathbf{A}_{k2}\mathbf{Z}^{*2}\cdots+\mathbf{A}_{kk}\mathbf{Z}^{*k} +\cdots+\mathbf{A}_{k(N-1)}\mathbf{Z}^{*(N-1)}+\mathbf{A}_{kN}\mathbf{Z}^{*N}=\mathbf{D}_k\mathbf{Z}^o, \quad (19)$$

where $\mathbf{A}_{kk}$, $\mathbf{A}_{ki}$, and $\mathbf{D}_k$ are defined by the following equations:

$$\mathbf{A}_{kk}=\left\{\left(\mathbf{K}_f^k-\mathbf{K}_m\right)\left[\left(1-f_k\right)\mathbf{S}+f_k\mathbf{I}\right]+\mathbf{K}_m\right\} \quad (20)$$

$$\mathbf{A}_{ki}\left(i\neq k\right)=f_i\left(\mathbf{K}_m-\mathbf{K}_f^k\right)\left(\mathbf{S}^i-\mathbf{I}\right) \quad (21)$$

$$\mathbf{D}_k=\left(\mathbf{K}_m-\mathbf{K}_f^k\right). \quad (22)$$

By extending Eq. (19) to all fillers, all eigentemperature gradient vectors can be expressed in the partitioned matrix form as

$$\begin{bmatrix}\mathbf{A}_{11} & . & . & . & \mathbf{A}_{1N}\\ . & . & . & . & .\\ . & . & . & . & .\\ \mathbf{A}_{N1} & . & . & . & \mathbf{A}_{NN}\end{bmatrix}\begin{bmatrix}\mathbf{Z}^{*1}\\ .\\ .\\ \mathbf{Z}^{*N}\end{bmatrix}=\begin{bmatrix}\mathbf{D}_1\\ .\\ .\\ \mathbf{D}_N\end{bmatrix}\mathbf{Z}^o, \quad (23)$$

which is further reduced into a simplified form as follows:

$$\mathbf{A}\mathbf{Z}^*=\mathbf{D}\mathbf{Z}^o. \quad (24)$$

The eigentemperature gradient vector of each layer is determined by the equation

$$\mathbf{Z}^*=\mathbf{A}^{-1}\mathbf{D}\mathbf{Z}^o=\mathbf{F}\mathbf{Z}^o. \quad (25)$$

By using Eq. (12), the temperature gradient vector $\mathbf{Z}_c$ in the composite is expressed by

$$\mathbf{Z}_c=(1-f)\mathbf{Z}_m+\sum_{k=1}^{N} f_k\mathbf{Z}_f^{*k}=\mathbf{Z}^o+\sum_{k=1}^{N} f_k\mathbf{F}_k\mathbf{Z}^o. \quad (26)$$

The thermal conductivities of the laminated composite of N layers can be finally given by

$$\mathbf{K}_c=\mathbf{K}_m\left[I+\sum_{k=1}^{N} f_k\mathbf{F}_k\right]^{-1}. \quad (27)$$

2.3 Conventional approach for thermal conductivities of laminated composites

It is well known that the thermal conductivity of a unidirectional lamina parallel to the filler direction, k_L, is obtained from the rule of mixtures. Many attempts, however, have been made to derive the thermal conductivity of the lamina perpendicular to the filler direction, k_T, and their basic concepts are classified into the potential theory approach and the electrical resistance analogy [6]. Rolfes and Hammerschmidt [6] concluded that the self-consistent formula based on the potential theory approach gives the most realistic result. These conductivities for the lamina are expressed as

$$k_L = (1-f)k_m + fk_f \ , \tag{28}$$

$$k_T = \left[\frac{(1-f)k_m + (1+f)k_f}{(1+f)k_m + (1-f)k_f}\right]k_m \, , \tag{29}$$

where k_m, k_f, and f denote the thermal conductivities of the matrix and filler and the filler volume fraction, respectively. The global thermal conductivities of the lamina with filler orientation angle θ_i with respect to x_3 are then derived as

$$k_1 = k_L \sin^2\theta_i + k_T \cos^2\theta_i \, , \tag{30}$$

$$k_3 = k_L \cos^2\theta_i + k_T \sin^2\theta_i \, . \tag{31}$$

The global thermal conductivity of the lamina in the thickness direction is the same as Eq. (29). Finally, the global thermal conductivities of the laminated composite along the global coordinate system, K_{11}, K_{22}, K_{33}, are computed by using an electrical analogy, which are represented as

$$K_{33} = \sum_{i=1}^{n} h_i k_{3i} \Big/ h \, , \tag{32}$$

$$K_{11} = \sum_{i=1}^{n} h_i k_{1i} \Big/ h \, , \tag{33}$$

$$K_{22} = h \Big/ \sum_{i=1}^{n} \frac{h_i}{k_{ti}} \, , \tag{34}$$

where h, h_i, and subscript i represent the total thickness of the laminated composite, thickness of the i-th lamina, and the i-th lamina, respectively.

3. Results and discussion

3.1 Thermal conductivities of composites with penny-shaped fillers

Let's consider a composite with penny-shaped fillers whose axes coincide with the global coordinate system, as shown in Fig. 2. The thermal conductivities of the matrix and filler are denoted by k_m and k_f, respectively. Since all fillers are aligned with the global coordinate system, the thermal conductivities of the composite with misoriented fillers expressed as Eq. (15) in section 2.1 can be further simplified by excluding the orientation effect of the fillers. The transformation matrix **X** is reduced to the identity matrix, so the matrix **T** is further simplified. The resulting thermal conductivities of the composite are given by

$$\mathbf{K}_c = \mathbf{K}_m\left[\mathbf{I} + f\mathbf{T}\left(\mathbf{K}_m - \mathbf{K}_f\right)\right]^{-1} \, , \tag{35}$$

where $\mathbf{T} = \left\{\left(\mathbf{K}_f - \mathbf{K}_m\right)\left[(1-f)\mathbf{S} + f\,\right] + \mathbf{K}_m\right\}^{-1}$. (36)

The thermal conductivities in x_1 and x_3 directions are the same and easily computed because all matrices in Eqs. (35) and (36) are diagonal. The thermal conductivity in these directions, k_L, can be obtained from Eq. (35) as

$$k_L = \frac{k_m}{1 + f\,T_{33}\left(k_m - k_f\right)} \, . \tag{37}$$

The 33 component of the matrix **T** is derived from Eq. (36) and as follows:

$$T_{33} = \frac{1}{\left(k_f - k_m\right)\left[(1-f)S_{33} + f\,\right] + k_m} \, , \tag{38}$$

where S_{33} represents the 33 components of the Eshelby tensor for the penny-shaped filler. By inserting Eq. (38) into Eq. (37), k_L is written as

$$k_L = \left[1 + \frac{f\left(k_f - k_m\right)}{(1-f)S_{33}\left(k_f - k_m\right) + k_m}\right]k_m \, . \tag{39}$$

Eshelby tensors of various filler shapes for the heat conduction problem are summarized in the literature[1] and S_{33} for the penny shape shown in Fig. 2 (c) is given by

$$S_{33} = \frac{\pi}{4}\frac{a_2}{a_3} \, . \tag{40}$$

As a limiting case of the penny shape, let's consider a_2/a_3 to be 0, which means that its thickness is negligible compared with its diameter. Then, S_{33} is equal to 0. Therefore, the thermal conductivity of the composite is finally represented as

$$k_L = (1-f)k_m + fk_f \,, \tag{41}$$

which is equivalent to the result of the composite with layers of the components by the parallel model.

Following the same procedures mentioned in the above, the thermal conductivity in x_2 direction, k_T, is represented as

$$k_T = \left[1 + \frac{f(k_f - k_m)}{(1-f)(k_f - k_m)S_{22} + k_m}\right] k_m \,, \tag{42}$$

where the Eshelby tensor for the penny shape is $S_{22} = 1 - \pi a_2/2a_3$. As a_2/a_3 goes to 0, S_{22} approaches to 1. Therefore, the thermal conductivity in this direction gives rise to

$$k_T = \frac{k_m k_f}{fk_m + (1-f)k_f} \,, \tag{43}$$

which is equivalent to the result of the composites with layers of the components by the series model. It is obvious from Eqs. (41) and (43) that the composite with penny-shaped fillers of $a_2/a_3 = 0$ simulates exactly layers of the components aligned either perpendicular or parallel to the heat flow like the laminated composite, so a lamina can be treated as a penny-shaped filler.

3.2 Thermal conductivity of laminated composites

Since MEM applied to the composites with the penny-shaped fillers can simulate exactly the laminated structures, it can be further extended to predict the thermal conductivities of the laminated composites using the results of multi-phase composites shown in section 2.2. A nonsymmetric laminated composite with five laminae is selected for comparison, whose filler orientation angles are 0, 10, 20, 30, and 40 degrees. Its schematic representation for laminated composite with three laminae is shown in Fig. 3. All laminae are assumed to be identical in thickness and also in filler volume fraction. Each lamina has different thermal conductivity and it is computed from Eq. (15) shown in section 2.1. The laminae with homogenized thermal conductivities are considered as the penny-shaped fillers and again embedded into the matrix material, as shown in Fig. 2. By using MEM [16, 17] shown in section 2.2, the thermal conductivity of the laminated composite is computed. The results of K_{11}, K_{22}, and K_{33} predicted by the present model are compared with those by the conventional approach [6, 8, 15] with the potential theory and electrical analogy and shown in Fig. 4, where the filler volume fraction means the filler volume fraction of the lamina. The results are plotted only for a thermal conductivity ratio k_f/k_m of 10 because the results for different conductivity ratios by the two methods are the same. It is shown from Fig. 4 that both results are exactly the same for various filler volume fractions and thermal conductivity ratios. It can be concluded that the present model can be applied to predict the thermal conductivities of the laminated composites.

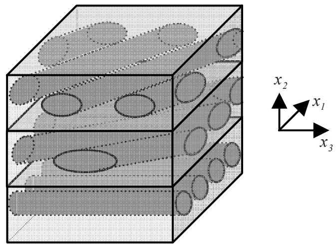

Fig. 3. Schematic representation of a laminated composite with three laminae with different orientation angles of fillers.

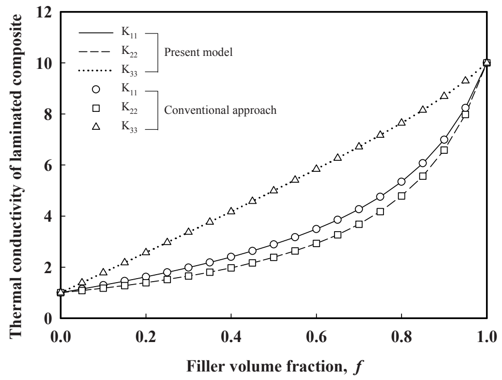

Fig. 4. Comparison of thermal conductivities of laminated composite predicted by the present model and conventional approach as a function of filler volume fraction, where thermal conductivity ratio k_f/k_m is 10.

4. Conclusions

The modified Eshelby method has been applied to analyze the thermal conductivities of the composite with penny-shaped fillers and its results for the penny shape of the negligible thickness are shown to be exactly the same as the results by the series and parallel models for composites with layers of the components. These results are extended to predict the thermal conductivities of the laminated composites. The thermal conductivities of laminae are computed and they are replaced by the penny-shaped fillers. The laminated composites are finally considered as composites composed of the matrix and the penny-shaped fillers for applying the modified Eshelby method. The thermal conductivities of the laminated composites for various filler volume fractions and thermal conductivity ratios predicted by the present model are consistent with the results by the conventional approach combined with the potential theory and electrical analogy. It is proved through the present study that the present model can be applied to predict the thermal conductivities of the laminated composites.

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